

Contorted Flavors in Grand Unification and Proton Decay

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Conventionally we collect electron and up/down-quarks $(u, d) \oplus (\nu_e, e)$ to form unified generations, and collect the left-handed and the right-handed fermions of the same flavors $(\nu_e, e) \oplus \bar{e}$. We can alternatively have a contorted multiplet, made by pairing different quarks and leptons, like $(u, d) \oplus (\nu_\mu, \mu)$ or $(u, d) \oplus (\nu_\tau, \tau)$, and/or between different left and right handed fermions to $(\nu_e, e) \oplus \bar{\mu}$, etc. These can suppress proton decay, due to its high flavor dependence, while having the correct fermion masses.

Unification of apparently different forces of Nature into a simpler one has driven the history of physics. With the unification of weak and the electromagnetic forces, the establishment of the Standard Model of particle physics (SM) is based on the gauge theory of the strong and the electroweak interactions described by the group $SU(3)_C \times SU(2)_L \times U(1)_Y$. A further unification was considered and resulted in Grand Unified Theory (GUT) [1–3], possibly with the aid of the supersymmetry (SUSY), which have been the best hint for the possible single nature of strong and electroweak forces. Being gauge interaction, extending the gauge groups into a (semi-)simple embracing group naturally describes a unified force. For the size of unification scale, the running of gauge coupling is the most important hint on the gauge sector [4]. Besides the Grand Unification scheme at the field theory level, every known top-down approach, for instance string theory, has the same structure of a larger simple group and larger representations [5]. Thus the embeddability of quantum numbers are quite compelling.

Consequently the transforming matters are also unified to yield larger one(s). Thus the extension fixes relative identities between the quark and the lepton families. Upon forming unified multiplets, conventionally the following is assumed:

1. Quarks (color triplets) and leptons (color singlets) form the same unified generations according to increasing order of masses.
2. Left-handed (weak doublet) and right-handed (weak singlet) fermions of the same flavor form the same unified generations.

Although conventional, there is *no a priori reason to follow these assumptions*. By relaxing the first assumption we can consider

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \oplus \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \oplus \begin{pmatrix} u \\ d \end{pmatrix} \text{ or } \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \oplus \begin{pmatrix} u \\ d \end{pmatrix}, \quad (1)$$

and so on. For example, in one ‘contorted’ version of $SU(5)$ model may contain $\bar{\mathbf{5}} = (\bar{d}_i, \nu_\tau, \tau)$ [24]. By relaxing the second assumption we can have

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \oplus \bar{e} \rightarrow \begin{pmatrix} \nu_e \\ e \end{pmatrix} \oplus \bar{\mu}, \quad (2)$$

and so on. In an extreme case, one unified generation can mix all the quarks and leptons $\bar{\mathbf{5}} = (\bar{d}_i, \nu_\mu, \tau)$, $\mathbf{10} = (\frac{c_i}{s_i}, \bar{t}_i, \bar{e})$, for instance. No physical principle restricted such pairing, such as anomaly cancellation, showing each generation is completely closed even if we consider different pairing. For a formation as in (1), we will see that the mass hierarchy can be a guideline.

FLAVOR STRUCTURE

After electroweak symmetry breaking, we diagonalize the flavor eigenstates to the mass eigenstates

$$\begin{aligned} \psi_F &\rightarrow U_F^\dagger \psi_F, & \bar{\psi}_F &\rightarrow W_F \bar{\psi}_F, \\ Y_F &\rightarrow Y_F^{\text{diag}} = U_F Y_F W_F^\dagger, & F &= U, D, N, E, \end{aligned} \quad (3)$$

with obvious notations [25]. In SM, we *define* the electron as the lightest charged lepton, thus its flavor and mass eigenstates are identical. At the same time we independently define the up quark as the lightest quark, because it is not connected to leptons by any interactions. One may have equal right to define the muon as the lightest lepton, in SM this is just renaming. However in GUT, the quarks and leptons are related, for instance e is related to u, \bar{u} in $\mathbf{10}$ of $SU(5)$. Once we define the electron by fixing e and \bar{e} in the entire multiplets $\mathbf{10}, \bar{\mathbf{5}}$, the up-type quarks follow from the relative mass eigenstates u', \bar{u}' by

$$u = W_L U_U^\dagger u', \quad \bar{u} = U_L^\dagger W_U \bar{u}', \quad \text{with respect to } \bar{e}, \quad (4)$$

to form the flavor multiplet. Therefore defining the muon as the lightest lepton modifies the identity of the up quark as the lightest, since the new definition is now distinguished. Thus in general, even the flavor eigenstates of up-type quarks are not guaranteed to be mass eigenstates. Also it means that we can distinguish contorted flavors, as in (1) and (2). On the other hand, this information is not relevant in the low energy physics, since there is no interaction connecting like (4) in SM. Even if a low level theory predicts the lighter mass for the muon than the electron belonging to $(\frac{u}{d} \bar{u} \bar{e})$, misidentification of the lightest as electron *does not* change SM.

After fixing the electron, the relative fermions in $SU(5)$ are completely connected by the relative basis change matrices (RBCMs), among them are CKM matrix $V_{UD} \equiv$

$U_U^\dagger U_D$ [7] and PMNS matrix $V_{EN} \equiv U_E^\dagger U_N$ [8], up to renormalization effects. For example, the emission of $Y^{1/3}$ boson from down quark to *conjugate*-up quark accompanies the RBCM $V_{d\bar{u}} \equiv (U_D^\dagger W_U)_{d\bar{u}}$, which is to be distinguished from the CKM element $V_{du} \equiv (U_D^\dagger U_U)_{du}$.

In the minimal $SU(5)$, we have two Yukawa couplings $Y_U = Y_U^T$ and $Y_D = Y_E^T$ diagonalized by four matrices $U_U, W_U, U_D = U_E, W_D = W_E$. We can fix $U_U = V_{UD} = V_{CKM}$ and $W_U = V_{CKM}^T P^T$, where P is a diagonal $SU(3)$ matrix. Thus, besides known quark and lepton masses and CKM matrix, we have two new phases [6]. Since $SO(10)$ GUT completely relates all the fields in SM, we can in principle completely observe the Yukawa coupling itself, from interactions in GUT. This means, it is not possible to arbitrarily align the flavor and the mass eigenstates.

CONTORTED FLAVORS

In the minimal $SU(5)$, the RBCM between down-type quarks and conjugate leptons is $V_{D\bar{E}} = U_D^\dagger W_E = \mathbf{1}$, without loss of generality. It means that, for example by charged $X^{4/3}$ boson exchange, d always transits to \bar{e} , $V_{d\bar{e}} = 1$, but there is no transition between d and $\bar{\mu}$, $V_{d\bar{\mu}} = 0$. It is because there is no difference in mass diagonalization between down-type quarks and leptons $U_E = U_D, W_E = W_D$.

However the minimal models lead to bad mass relations $m_e/m_\mu = m_d/m_s$, which is renormalization group invariant. One should introduce more Higgs fields and as many complex Yukawa matrices. Usual fermion textures in non-minimal GUT imply similar RBCMs between quarks and leptons, with slightly modified diagonality, but in general there emerges many new phases.

To correct the relations, the main paradigm after Ref. [15] has been to introduce 45 and higher dimensional Higgses. For illustration, we adopt the most general renormalizable Yukawa coupling including **45**: **10**·**10**·**5_H** + **10**· **$\bar{5}$** · **$\bar{5}_H$** + **10**· **$\bar{5}$** ·**45_H**, where the coefficients are in general complex matrices in the flavor basis. With some assumptions on the form of Yukawa couplings and VEV, we can have the following form of mass matrices for isospin $-1/2$ fermions

$$M_d = \begin{pmatrix} l-m & a & 0 \\ a & k+c & 0 \\ 0 & 0 & b \end{pmatrix}, \quad M_l = \begin{pmatrix} l+3m & a & 0 \\ a & k-3c & 0 \\ 0 & 0 & b \end{pmatrix}, \quad (5)$$

which is also a typical texture from ‘minimal renormalizable’ $SO(10)$, making use of **10** and **126** Higgses [16]. The elements in the same letters come from the same VEVs of **5** and **45**, with some Clebsch-Gordan coefficients. If we take $l \sim m \sim k \sim 0$, in the parameter range $|a| \ll |c| \ll |b|$, we have mass relations $m_d \simeq 3m_e, m_s \simeq m_\mu/3, m_b \simeq m_\tau$ after diagonalization

[15]. However in the parameter range $|l-m| \ll |c+k|$ and $|k-3c| \ll |l+3m|$, with small a and large b as before, we have *inverted lepton mass hierarchy* with respect to quarks.

$$m_d < m_s < m_b, \quad 'm_\mu' < 'm_e' < m_\tau. \quad (6)$$

However we note that, although lepton mass hierarchy is inverted, still *we identify the electron as the lightest charged lepton in SM*. Therefore it will be more appropriate to collect the multiplet as $(u, d) \oplus \mu$ and $(c, s) \oplus e$. If the parameters are aligned to fit the observed fermion masses $a^2/(l+3m) \simeq 0.5 \text{ MeV}/c^2$, $l+3m \simeq 106 \text{ MeV}/c^2$ (neglecting renormalization corrections), we cannot see the change in SM.

Of course, this has distinguished consequences in GUT, in particular proton decay that we see shortly. Because we have inverted hierarchy of the charged leptons, we have *mostly off-diagonal* U_E and W_E with respect to the one with normal hierarchy,

$$U_E \rightarrow U_E T_{12}, \quad W_E \rightarrow W_E T_{12}, \quad Y_E^{\text{diag}} \rightarrow T_{12} Y_E^{\text{diag}} T_{12}^\dagger, \\ T_{12} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in S_3 \subset SO(3). \quad (7)$$

Now there is no transition between d and \bar{e} , $V_{d\bar{e}} \sim 0$ by $X^{4/3}$ boson exchange, but there is one between d and $\bar{\mu}$, $V_{d\bar{\mu}} \sim 1$. In more general contortions, T can be any element of permutation symmetry of order 3, S_3 , which is a discrete subgroup of $SO(3)$.

In string compactifications, gauge bosons and matter fermions have similar embedding as GUT [5, 17]. However we do not break them by Higgs mechanism, but by projection conditions associated with symmetries of internal manifold, just leaving footprints of unified multiplets. The fermion mass hierarchy is determined by geometry [18], from distributions of fields in the internal spaces. Thus we can have large suppression for down-type quarks while having small suppression for leptons in the same multiplet and vice versa. Thus the contortion naturally arises in string theory.

Because of symmetric Yukawa coupling, we cannot contort u sector with respect to \bar{u} in $SU(5)$. Also u and d sectors are related by CKM matrix, so we have a similar hierarchy between up and down type quarks. We can further generalize the idea, as in (2), by exchanging *only the right-handed* fermions between different generations, $\begin{pmatrix} \nu_e \\ e \end{pmatrix} \oplus \begin{pmatrix} u \\ d \end{pmatrix} \oplus \bar{\mu}$ and $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \oplus \begin{pmatrix} u \\ d \end{pmatrix} \oplus \bar{e}$, or $\begin{pmatrix} u \\ d \end{pmatrix} \oplus \bar{t} \oplus \bar{d}$ and $\begin{pmatrix} t \\ b \end{pmatrix} \oplus \bar{u} \oplus \bar{b}$. Then we have more freedom for quark and lepton masses. In particular in $SU(5)$, highly asymmetric texture is possible, thus the ‘minimal’ $SU(5)$ can be different. For example if we exchange \bar{u}

and \bar{c} to form a contorted family, the original symmetry constraint from $\mathbf{10}_i \cdot \mathbf{10}_j$ on $Y_{u\bar{c}} = Y_{c\bar{u}}$ becomes

$$Y_{u\bar{u}} = Y_{c\bar{c}}. \quad (8)$$

PROTON DECAY BY DIMENSION SIX OPERATORS

For the association of quark and lepton generations, the only direct experiment have been the proton decay [9]. First we consider the effect of dimension six operators. The typical proton decay amplitude is $p \rightarrow e^+\pi^0$ mediated by X, Y bosons [10],

$$\Gamma = \alpha^2 \frac{m_p}{64\pi f_\pi^2} (1 + D + F)^2 \left(\frac{g_5^2 A_R}{M_{\text{GUT}}^2} \right)^2 \times \left(|V_{u\bar{u}} V_{e\bar{d}}^*|^2 + |V_{u\bar{u}} V_{d\bar{e}}^* + V_{d\bar{u}} V_{u\bar{e}}^*|^2 \right). \quad (9)$$

Here α is the low energy contribution from hadronic matrix element, can be calculated by lattice QCD [14]. m_p, f_π, g_5, A_R are respectively the proton mass, the pion decay constant, the unified gauge coupling, the renormalization factor. $D \simeq 0.76$ and $F \simeq 0.48$ are interaction strength between baryons and mesons obtained from chiral perturbation theory. Neglecting the last factor, the partial lifetime for SUSY case is

$$\tau/B = \Gamma^{-1} \simeq 8 \times 10^{34} \text{ years} \times \left(\frac{0.015 \text{ GeV}^3}{\alpha} \right)^2 \left(\frac{M_{\text{GUT}}}{10^{16} \text{ GeV}} \right)^4 \times (\text{flavor factor}). \quad (10)$$

The gauge boson mass is guided by running gauge coupling: $M_{\text{GUT}} \sim 10^{15} \text{ GeV}$ for non-SUSY and $M_{\text{GUT}} \simeq 3 \times 10^{16} \text{ GeV}$ for SUSY cases [13]. Considering a typical decay channel like $p \rightarrow e^+\pi^0$, we have the partial mean lifetime, in non-SUSY case $\tau/B \sim 10^{31-35} \text{ years}$, depending on extensions. The observed bound for the proton lifetime is $\tau/B > 5 \times 10^{33} \text{ years}$ [13].

The last factor contains RBCMs, which are respectively the components of $U_U^\dagger W_U, (U_E^\dagger W_D)^\dagger, U_U^\dagger W_U, (U_D^\dagger W_E)^\dagger, U_D^\dagger W_U, (U_U^\dagger W_E)^\dagger$, all of which lie outside CKM and PMNS. The amplitude is highly flavor dependent [6, 11]. In the minimal $SU(5)$, all the elements are set to identity as above, except $V_{U\bar{U}} = P_{CKM} P, V_{D\bar{U}} = V_{CKM}^T P^T, V_{U\bar{E}} = V_{CKM}^T P^T V_{CKM}$, so the flavor factor is nearly 5. The flipped $SU(5)$ [12] contains only the first term in the last factor [19].

If we contort the first two generations as in (7), then we have $V_{Q\bar{L}} \simeq T$ with $V_{u,d\bar{e}} \sim 0, V_{u,d\bar{\mu}} \sim 1$. Thus the transition from quark to lepton is dominated by off-diagonal elements. Eventually the proton decay is dominated by muon. Consequently from (9) there can be no $p \rightarrow e^+\pi^0$, but *opens proton decay channels to other leptons*, like

$$p \rightarrow \mu^+\pi^0, \quad (11)$$

with the same lifetime as the original case. The bound for this process is $\tau/B > 3.7 \times 10^{33} \text{ years}$, is almost same order of magnitude to proton decay into positron. Thus large off-diagonal RBCM element does not help much.

There have been only water (e.g. Super-Kamiokande) and earthly metal (e.g. iron in Soudan II) based experiments, thus are able to identify decay from protons and neutrons, consisting of up and down quarks as initial states. So if the electron is connected with other quarks than u, d , we cannot observe the decay from other sources.

The pairing $(u, d) \oplus \tau$ is achieved if we invert the first and the third generation leptons. The outgoing state would be $ud \rightarrow \tau^+\bar{u}$, instead of $ud \rightarrow e^+\bar{u}$. In this case the proton cannot decay into tauon, since it is heavier ($m_\tau = 1777 \text{ MeV}/c^2$) than proton ($m_p = 938 \text{ MeV}/c^2$). Thus if the tauon component of the mixing is high, we will lose decay information for $p \rightarrow e^+\pi^0$. However still there are decay channels

$$p \rightarrow \bar{\nu}\pi^+ \quad \tau/B > 2.5 \times 10^{31} \text{ years}, \quad (12)$$

$$n \rightarrow \bar{\nu}\pi^0 \quad \tau/B > 1.1 \times 10^{32} \text{ years}, \quad (13)$$

where we cannot distinguish the flavor of neutrino, since we only seek missing energy. Notably this decay does not exist in the flipped $SU(5)$.

In some model, with contortion like (2), we can suppress some nucleon decay by dimension six operators, however it is very hard to overcome the constraint from (12) and (13). It is because, exchanging right-handed quarks and leptons, there is always a pair $\bar{u} \oplus e$ in almost all known unification models. Thus a nucleon decay into neutrino is very important verification of all kind of GUT interactions [20].

We may make theory with just simple exchange $e \leftrightarrow \mu$ to have a generation $(u, d) \oplus \mu$, without mass inversion. Then we can easily see that, since the RBCM does not change $V_E = W_E = \mathbf{1}$, so that proton always decays into the lightest lepton, whatever name it has. Thus we have no change.

PROTON DECAY BY DIMENSION FIVE OPERATORS

Although the dimension six operators are not so harmful in SUSY GUT, there are relevant dimension five operators from integrating out the triplet Higgs pair, known as L^4 and R^4 : $c_{5L}^{ijkl} Q_i Q_j Q_k L_l / (2M_T) + c_{5R}^{ijkl} \bar{u}_i \bar{d}_j \bar{u}_k \bar{e}_l / M_T$, where

$$c_{5L}^{ijkl} = (Y_D^{\text{diag}})^{km} (V_{QL})_m^l (Y_U^{\text{diag}})^{in} (V_{QQ}^\dagger)_n^j, \quad (14)$$

$$c_{5R}^{ijkl} = (Y_D^{\text{diag}})^{mj} (V_{D\bar{U}})_m^i (Y_U^{\text{diag}})^{nl} (V_{U\bar{E}}^\dagger)_n^k, \quad (15)$$

at the unification scale. Here M_T is the triplet Higgs mass, obtained by unification condition. Because of

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- [24] All the fermions here are left-handed Weyl spinors, with the charge conjugations for the barred letters. i is color index.
- [25] We will use upper case subscripts for matrices, and lower cases for their components. For example, Y_e^{diag} is the (11)-component of Y_E^{diag} and $V_{u\bar{e}}$ is (11)-component of the matrix $V_{U\bar{E}} \equiv U_E^\dagger W_L$, etc.